Discussion and Conclusions

A step-size control procedure based on an estimate of the truncation error of an embedded second-order method has been presented for the classical fourth-order Runge-Kutta method. For the sample problem, this procedure has been shown to be more efficient than the method of doubling as applied by Gear.[7]

References

[1] Fehlberg, E., "Low-Order Classical Runge-Kutta Formulas with Step-Size Control and Their Application to Some Heat Transfer Problems," NASA TR R-315, 1969.

[2] Fehlberg, E., "Classical Fifth-, Sizth-, Seventh-, and Eighth-Order Runge-Kutta Formulas with Stepsize Control," NASA TR R-387, 1968.

[3] Shampine, L. F. and Gordon, M. K., *Computer Solution of Ordinary Differential Equations*, W. H. Freeman and Co., San Francisco, Calif., 1975.

[4] Shampine, L. F. and Watts, H. A., "Comparing Error Estimators for Runge-Kutta Methods," *Mathematics of Computation*, Vol. 25, July 1971, pp. 445-455.

[5] Lapidus, L. and Seinfeld, J. H., *Numerical Solution of Ordinary Differential Equations*, Academic Press, New York, 1971.

[6] Ralston, A., "Runge-Kutta Methods with Minimum Error Bounds," *Mathematics of Computation*, Vol. 16, 1962, p. 431.

[7] Gear, C. W., *Numerical Initial Value Problems in Ordinary Differntial Equations*, Prentice-Hall, Englewood Cliffs, N.J., 1971.

Some Laser Velocimeter Measurements in the Turbulent Wake of a Supersonic Jet

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Introduction

ALTHOUGH the phenomena of turbulent jet wakes have been studied for a long time, many fundamental questions have not been answered. A large portion of turbulent wake data has been obtained by pitot static probes or by hot-wire anemometers. Unfortunately, for high subsonic, transonic, and supersonic flow, hot-wire anemometers and pitot probes are difficult to use. The laser velocimeter (LV) has been publicized as one remedy to these difficulties.[1,2]

Previous examinations of turbulent jet wakes were directed primarily toward subsonic flows.[3,4] Eggers[5] and others have measured supersonic jet wakes with pitot probes. Conventional hot-wire data are available for most of the subsonic flows, including energy spectra, spatial correlations, and turbulent shear stresses. However, for the supersonic flows only mean axial velocities previously were available. More recently, Avidor[6] and others used laser velocimeters to measure the turbulent fields of compressible jets. These LV investigations, however, were not aimed at the flow data but were meant to be investigations of laser velocimeters. As a result, complete turbulence data and correlations are not available.

In the present study, the near wake of a supersonic jet with a Mach number of 2.22 is examined with a LV. Mean flow data are found and compared to previously obtained supersonic pitot probe data. Turbulence intensities are found and compared to previously obtained subsonic hot-wire values. Differences between the present LV data and previous data are discussed. The purpose of this Note is therefore twofold. First, potential sources of previously unaddressed LV biases are brought to the attention of other experimenters. Second, unbiased data are presented for a portion of supersonic jet wake where Mach number effects are important.

Experimental Apparatus and Procedure

The experimental investigation was conducted utilizing an air blowdown system and a converging-diverging axisymmetric nozzle with an exit diameter ($D=2r_e$) of 25.78 mm and a designed exit Mach number of 2.22.[5,7] The system was operated at a stagnation pressure in a plenum tank which was chosen to match the jet exit pressure to the ambient atmospheric pressure. The LV used was of the individual realization dual-beam forward scatter type. A 5-W Coherent argon-ion laser was used for a one-component system. The effective probe volume size was approximately 176 μ diam and approximately 1500 μ long. Frequency shifting by the use of a Bragg cell or other technique was not used in the present investigation. The entire optical system was mounted on a large mill table capable of traversing the flow in three directions, and the positioning accuracy was better than 0.1 mm.

As a particle passes through the probe volume, light is scattered, collected by the receiving optics, and focused onto a photomultiplier tube (PMT). After the low-frequency pedestal voltage was removed from each PMT signal by the use of a bandpass filter, each Doppler signal was analyzed by a zero-crossing burst-type four- and eight-count comparison processor.[7] Dioctyl phthalate (DOP) particles 0.5-2 μ in diameter were generated with a Laskin nozzle with impactor plates. In previous inviscid steady internal transonic flow measurements,[8] the LV was found to operate very accurately. The average velocity LV data generally were within 2% of static pressure data, and the LV signal processor was found to operate with a very low noise level (measured "turbulence intensities" less than 0.01).

With an individual realization laser velocimeter, such as the one utilized in the present study, the output of the processor is not continuous. Therefore, if the sample rate is not high enough to insure that a significant number of samples are being measured within a temporal interval less than the macro time scale, obtaining energy spectrum data or length or time-scale data, as with a hot-wire anemometer, is not possible. Smith and Meadows[9] and Mayo et al.[10] have demonstrated that with high enough sample rates obtaining spectral data is possible. However, with high-speed flows as presented in this Note, seeding the flow densely enough is not possible to obtain this high sample rate. Thus, the signals were treated in velocity histograms, with other quantities of interest calculated therefrom, including the mean velocity and turbulence intensity. Only the near-wake region of the supersonic turbulent jet was "probed" in the present investigation. Axial velocity data were obtained for five axial positions X. At each axial position, velocity histograms were recorded for 15 to 20 radial positions r. Approximately 1500 data samples were obtained for each position.

Results and Discussion

Laser Velocimeter Biases

Data from the velocity histograms have been analyzed and used to construct mean velocity profiles at each axial location. A typical plot is presented in Fig. 1 for $X/D=1.005$. Com-

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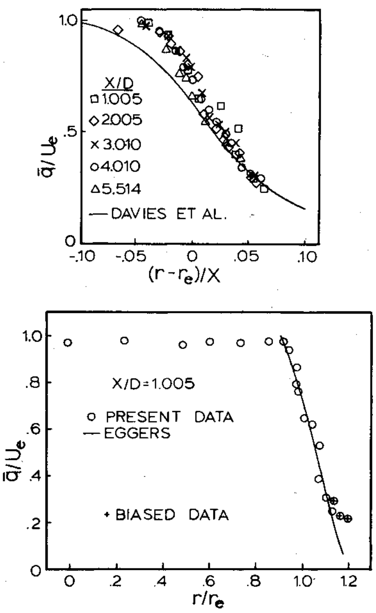

Fig. 1 Present axial component mean velocity profiles compared to Eggers[5] and to Davies et al.[3]

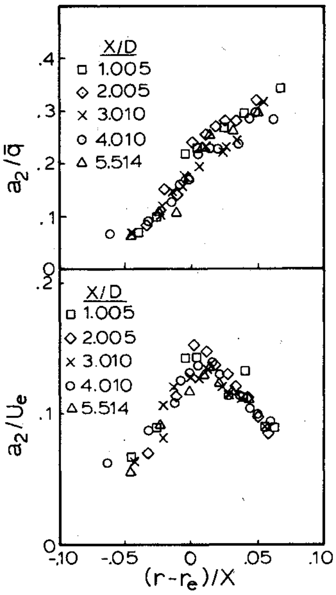

Fig. 2 Present axial component turbulence intensity profiles based on jet exit and local mean velocities.

parison is made to the pitot probe data of Eggers.[5] The present mean velocity data agree within 3% with Eggers' data for low values of r/r_e. In the region of higher turbulence and lower average velocity, complete and unbiasing pedestal removal became very difficult. Because of the high-pass frequency filters and the electronic signal processor, some of the signals representing slower particles were rejected. These data points are noted in Fig. 1. The resulting calculated average velocities were therefore higher than that reported by Eggers in these regions. For $X/D = 1.005$, this biasing occurred for approximate values of r/r_e of 1.10 and greater.

Four major reasons for signal biasing in the present work are acknowledged. First, when large turbulence intensities based on local mean velocities are encountered, complete and unbiasing pedestal removal from the individual signals becomes difficult and sometimes impossible if frequency shifting is not used, since the range of the Doppler frequencies is large. Second, as a particle passes through the probe volume, its velocity can change due to the turbulent fluctuations. In the signal processor used in the present work, a four- and eight-count comparison was made comparing the times required for a particle to traverse four and eight fringes. Significant fluctuations can cause the four- and eight-count comparator to reject valid signals of particles, particularly slower particles. Third, as a particle passes through the probe volume, it must cross at least eight fringes. If it does not cross eight fringes, the signal is invalidated. Again, these rejected signals are from predominantly slower particles. These last two biases have not been acknowledged previously. Flack[11] showed theoretically that these restrictions can be equally important and can bias the constructed velocity histograms significantly. The purpose here is to present experimental data that indicate that LV biases are present for highly turbulent flows. Finally, nonuniform seeding in the mixing region due to seeding only in the jet portion of the flow can lead to a biased velocity histogram.

Similar biasing due to the finite size of a LV probe volume has been examined theoretically and experimentally by Karpuk and Tiederman,[12] who attributed biases to three sources: incorrect averaging techniques and spatial variations of the mean and rms velocities across the probe volume. Their results show that the measured mean velocity is higher than the actual mean velocity of the flow in regions of high turbulence intensity. The present results confirm this trend but identify other biases as well.

All of the data discussed thus far were for the axial velocity component. Attempts also were made to obtain data at angles to the axial. However, when the off-axial data were examined vectorially, they were not consistent with the directly measured axial data. The off-axial components were determined to contain the same measurement biases described previously.

Flack[11] theoretically analyzed the possible magnitudes of these and seven other biases resulting from signal processing, the finite probe volume and fringe-spacing size, and the turbulence characteristics. One quantity that was found to be of primary importance is the turbulence intensity based on the local mean velocity in the component direction being measured, i.e., $a_2/\bar{q}$. Large biases were determined to be possible for measurements made at moderate angles to the mean flow (which decreases $\bar{q}$). The present experimental results confirm this trend.

Analysis of Unbiased Data

Since the raw data now have been examined and data containing LV biases have been identified, the unbiased data now are analyzed and compared to previous subsonic jet wake data to determine any differences between sub- and supersonic jet wakes. The similarity of the present flow is examined by plotting the axial component of nondimensionalized velocity $\bar{q}/U_e$, where U_e is the jet exit velocity, vs the similarity parameter $(r-r_e)/X$. This plot is presented in Fig. 1. The flow does exhibit a high degree of self-similarity in the near-wake region, since the nondimensionalized velocity profiles are almost identical for all five axial stations.

Also presented in Fig. 1 is the similarity mean velocity profile presented by Davies et al.[3] for Mach 0.30 and 0.45 jets. Complete correlation between the present data and that of Ref. 3 was not expected because of the significant difference between the present and previous Mach numbers. The most apparent difference between the two sets of data is the fact that the width of the mixing region is smaller for the supersonic jet than for a subsonic jet.

In the investigation of Eggers, turbulence intensities were not obtained for this jet. Therefore, one of the major objectives of the present work was to obtain the turbulence intensity distributions. In Fig. 2, a plot of the turbulence intensity (based on the exit velocity) a_2/U_e vs the similarity parameter $(r-r_e)/X$ is presented. Again, a high degree of similarity of the flow is exhibited for all five axial positions. Maximum values of this turbulence intensity were found to exist at a value of approximately 0.05 for $(r-r_e)/X$. Also in Fig. 2, a plot of the turbulence intensity based on the local mean velocity $a_2/\bar{q}$ is presented. For large radial distances, signal biasing occurred as described previously, and deter-

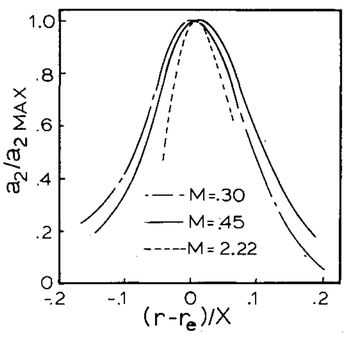

Fig. 3 Present normalized axial turbulence intensity profile compared to Davies et al.[3]

mining the resulting trend was not possible. This curve is significant in that it is consistent with the conclusion reached in the previous subsection. As the turbulence intensity (based on local measured mean velocity) is increased, signal biasing is increased. For the present data for all axial locations, local turbulent intensities above 0.3 led to significant differences between the present mean velocity data and those of Eggers.

Next, the present turbulence intensity data for the Mach 2.22 jet are compared to data for Mach 0.30 and 0.45 jets.[3] In Fig. 3, the nondimensionalized rms velocity (nondimensionalized by the maximum value) from a composite of Fig. 2 (a_2/U_e) is compared to data of Davies et al. The three sets of data are in qualitative agreement. However, since the Mach number of the present data is significantly greater than unity, quantitative agreement was not expected. The present turbulence intensity data are seen to define a smaller mixing region width for the larger Mach number jets. This conclusion is consistent with the mean velocity data presented in Fig. 1.

Conclusions

Agreement of the presented mean LV velocity data with the pitot tube data is excellent. Differences between the present data and previous subsonic turbulence intensity data are attributed to compressibility effects. The velocity and turbulence intensity profiles also exhibit a high degree of self-similarity in the near wake. These profiles indicate that the width of the mixing region for the supersonic jet is smaller than that of the subsonic jet.

Several forms of previously unacknowledged LV signal biasing also were identified, particularly for regions of the flow with large values of turbulence intensity. The present examination was limited to values of local turbulence intensity of 0.3 before biasing was evident.

Theoretical predictions have been made for the magnitudes of the individual biases.[11] These results will be presented in a subsequent paper. In conclusion, the authors would like to see more experimental data to identify the magnitudes of LV biases when highly turbulent flows are measured.

Acknowledgments

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References

[1] Thompson, H. D. and Stevenson, W. H. (eds.), *Proceedings of the Second International Workshop on Laser Velocimetry*, Purdue Univ., West Lafayette, Ind., March 27-29, 1974.

[2] Eckert, E. R. G. (ed.), *Minnesota Symposium on Laser Velocimetry Proceedings*, Univ. of Minnesota, Bloomington, Minn., Oct. 22-24, 1975.

[3] Davies, P. O. A. L., Fisher, M. J., and Barratt, M. J., "The Characterization of the Turbulence in the Mixing Region of a Round Jet," *Journal of Fluid Mechanics*, Vol. 15, Pt. 3, March 1963, pp. 337-367.

[4] Laurence, J. C., "Intensity, Scale, and Spectra of Turbulence in Mixing Region of a Free Subsonic Jet," NACA Rept. 1292, 1956.

[5] Eggers, J. M., "Velocity Profiles and Eddy Viscosity Distributions Downstream of a Mach 2.22 Nozzle Exhausting to Quiescent Air," NASA TN D-3601, Sept. 1966.

[6] Avidor, J. M., "Laser Velocimeter Measurements of Turbulence in a High Subsonic Jet," *AIAA Journal*, Vol. 13, June 1975, pp. 713-714.

[7] Thompson, H. D., Stevenson, W. H., Flack, R. D., and Zammit, R. E., "Laser Doppler Velocimeter Measurements in High Speed Flows," U. S. Army Missile Command Rept. RD-CR-75-2, Dec. 31, 1974.

[8] Flack, R. D. and Thompson, H. D., "Comparison of Pressure and LDV Velocity Measurements with Predictions in Transonic Flow," *AIAA Journal*, Vol. 13, Jan. 1975, pp. 53-59.

[9] Smith, D. M. and Meadows, D. M., "Power Spectra from Random-Time Samples for Turbulence Measurements with a Laser Velocimeter," *Proceedings of the Second International Workshop on Laser Velocimetry*, Purdue Univ., West Lafayette, Ind., March 27-29, 1974, pp. 27-46.

[10] Mayo, W. T., Jr., Shay, M. T., and Riter, S., "Digital Estimation of Turbulence Power Spectra from Burst Counter LDV Data," *Proceedings of the Second International Workshop on Laser Velocimetry*, Purdue, Univ., West Lafayette, Ind., March 27-29, 1974, pp. 16-26.

[11] Flack R. D., "The Application of a Laser Doppler Velocimeter in Interpreting Turbulent Structure," Ph.D. Thesis, Purdue Univ., West Lafayette, Ind., Dec. 1975.

[12] Karpuk, M. E. and Tiederman, W. G., Jr., "Effect of Finite-Size Probe Volume upon Laser Doppler Anemometer Measurements," *AIAA Journal*, Vol. 14, Aug. 1976, pp. 1099-1105.

On Near-Wall Collateral Flow in Skewed Turbulent Boundary Layers

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Introduction

ONE of the unresolved questions in skewed turbulent boundary layers concerns the existence of a collateral mean flowfield very close to the wall. Most of the existing data in the inner region of the boundary layer (particularly in the region extending up to $y^+ \approx 50$) seem to have been somewhat restricted by the size and/or response of the probes used, resulting in inadequate spatial resolution.[1-3] A careful examination of various experimental polar plots (see, for example, Ref. 4, Chap. 7) indicates near-wall collateral flow, i.e., in the inner region very close to the wall (sometimes extending up to local-to-freestream velocity ratios of as high as 0.5) the mean velocity vector does not change its direction. It should be noted that in most of these plots, the inner region between the wall and the apex was constructed by drawing a mean line through the origin and a few available data points (as low as two in some cases) near the apex.[1] However, recent measurements of Rogers and Head[5] using a specially designed hot-wire anemometer device showed skewed flow much closer to the wall; the data point closest to the wall corresponding to a resultant velocity ratio (local to freestream) of about 0.2. Similar trends are also observed in the later data of Vermeulen.[6] It is clear, therefore, that more (reliable) data are still needed to resolve experimentally the question of the existence or nonexistence of a near-wall collateral flowfield.

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